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On metastable vacua in perturbed $\mathcal{N} = 2$ theories

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Abstract

We study supersymmetry breaking in metastable vacua on the Coulomb branch of perturbed $\mathcal{N} = 2$ gauge theories, with gauge group $SU(2)$ and different matter content ($N_f = 0, 2, 4$). The theory is deformed with a superpotential which is a cubic polynomial in $u = \text{Tr } \Phi^2$, where Φ is the adjoint superfield. The allowed region of the perturbation parameters in this $\mathcal{N} = 1$ theory is plotted as a function of the moduli space coordinate. In the asymptotically free cases a significant fine-tuning in the perturbation parameters is needed to achieve metastable vacua in the weakly coupled region of the moduli space; a lower degree of fine-tuning is required in the strongly coupled regime. In the conformal case ($N_f = 4$ fundamentals) we find that also an explicit mass for the hypermultiplets must be introduced in order to generate metastable vacua. In the case of $N_f = 2$ fundamentals it is possible to achieve a metastable vacuum also in the neighborhood of the Argyres-Douglas fixed point (even if a large degree of fine-tuning is needed in this limit). Direct gauge mediation is discussed; gaugino masses of the same order of the SUSY-breaking can be obtained.

1 Introduction

Long-lived metastable vacua which break supersymmetry are generic in $\mathcal{N} = 1$ theories with massive fundamental matter, as was shown by ISS [1] (see [2, 3] for reviews of the topic). Usually the strong coupling of the models inhibits reliable calculations in models of dynamical symmetry breaking. In the ISS setting this is avoided using the Seiberg dual description [4], which is weakly coupled even in some cases where the original theory is strongly coupled. This is true in particular for $\mathcal{N} = 1$ SQCD with $N_c < N_f < 3/2 N_c$; in this regime the dual description is that of a $SU(N_f - N_c)$ gauge group with N_f fundamentals and some scalars. This theory is infrared free and computations are reliable, they lead to parametrically long lived meta stable states. The study of metastability inside the conformal window ($3/2 N_c < N_f < 3 N_c$) is more involved, also because the dual theory is not weakly coupled¹.

Another setting in which dynamical supersymmetry breaking in a long-lived metastable vacua is calculable is in $\mathcal{N} = 2$ theories, perturbed by a small superpotential. From the Seiberg-Witten curves [7, 8], the low energy effective theory on the Coulomb branch is exactly known, including the Kähler potential. Using this theoretical tool, in [9, 10, 11, 12] the issue of metastability was studied in $\mathcal{N} = 2$ theories perturbed by a superpotential². These vacua have been also realized in string/M-theory constructions in [14].

Consider the case of the $\mathcal{N} = 2$ theory with gauge group $SU(2)$, with arbitrary matter content consistent with asymptotic freedom; the Coulomb branch of the moduli space can be parameterized in this case by the coordinate $u = \text{Tr } \Phi^2$. The moduli space can be lifted by perturbing the theory with a small superpotential which is a function of u :

$$\mathcal{W} = \mu (u + \alpha u^2 + \beta u^3) . \quad (1)$$

The term linear in u is a mass term for the adjoint superfield; the terms proportional to u^2 and u^3 correspond instead to (dangerously) irrelevant operators. In [9] it was shown that for almost every point of the moduli space u_0 , it is possible to choose the coefficients (α, β) in (1) in such a way that a metastable vacuum is generated at $u = u_0$.

The direct computation of the superpotential which is needed to make each point of the moduli space metastable involves some rather cumbersome expressions for the $\mathcal{N} = 2$ prepotential (this is true especially in the cases with matter hypermultiplets). In [9] the allowed range of the deformation parameters allowing a metastable vacuum

¹ Recently examples of metastable vacua in the conformal window of $\mathcal{N} = 1$ SQCD (where some of the flavors are coupled to a gauge singlet) have been discussed in [5] and [6].

² Metastable vacua in $\mathcal{N} = 2$ theories perturbed by a Fayet-Iliopoulos term were studied in [13].

at the origin of the moduli space was explicitly computed in the pure Super-Yang-Mills case, for a generic number of colors. Explicit expressions for the case of a generic point in the moduli space in $\mathcal{N} = 2$ Super-Yang-Mills with gauge group $SU(2)$ were found in [12]. In this paper we compute numerically the allowed region for the deformation parameters (α, β) in some cases that were not discussed before.

In particular we focus on nearly scale invariant theories. Theories which are scale invariant do not have metastable states. This is true whether the symmetry is spontaneously broken or not. Using scale invariance, any candidate for a metastable state can be scaled to zero energy. In other words, no scale is available to produce the local stability around the metastable state. Scale invariant and conformal theories have many interesting properties not the least of them is the control on the value of vacuum energy [15].

The examples that we discuss for nearly scale invariant theories are the $\mathcal{N} = 2$ $SU(2)$ theory with $N_f = 4$ fundamental hypermultiplets and the $\mathcal{N} = 4$ theory. When the hypermultiplets are massless, it is not possible to generate a metastable vacuum at any point of the moduli space with the perturbation (1). The situation changes once one introduces a mass m for some of the hypermultiplets; then the result in [9] applies and it is possible to generate metastable vacua.

Another case that we study is the one with $N_f = 2$ fundamental massive hypermultiplets. For a critical value of the hypermultiplet mass, an Argyres-Douglas [16] conformal vacuum appears in the moduli space. It turns out that generating a metastable vacuum in the neighborhood of the conformal point is especially difficult, (α, β) must be rather fine-tuned. In this specific example we find that the allowed parameter range vanishes as $(u_0 - u_{AD})^3$, which is much stronger than nearby other supersymmetric vacua, where we find it vanishes as $(u_0 - u_{susy})$.

The examples that we consider in this paper contain flavor symmetries that can be gauged and coupled to external supersymmetric sectors, in order to realize direct gauge mediation (see for example [17, 3] for reviews). The gaugino masses obtained are of the same order of the SUSY-breaking. In particular, if we consider theories with zero mass term for the hypermultiplets, ordinary gauge mediation is realized.

The paper is organized as follows. In section 2 we review the theoretical setting and the general strategy to compute the range of parameters of (α, β) in order to generate a metastable vacuum. This gives a sense of the genericity of forming metastable states. In section 3 we discuss the $\mathcal{N} = 2$ Super-Yang-Mills theory ($N_f = 0$). In section 4 the conformal cases (the theory with $N_f = 4$ fundamentals and $\mathcal{N} = 4$ SYM) are studied. Section 5 is about the theory with $N_f = 2$ fundamentals which has conformal points. In all cases we search and find parametrically long lived metastable states. In section 6 we comment about direct gauge mediation. Section 7 contains

the conclusions. The appendix concerns the weakly coupled limit, where a compact analytical expression for the range of parameters of (α, β) can be found.

2 Theoretical setting

Consider an $\mathcal{N} = 2$ theory with gauge group $SU(2)$ and arbitrary matter content consistent with asymptotic freedom or conformal invariance. The moduli space can be parameterized by the VEV

$$u = \text{Tr } \Phi^2, \quad (2)$$

which spontaneously breaks the $SU(2)$ gauge symmetry to $U(1)$. The low-energy dynamics is described by the Seiberg-Witten curve [7, 8], which enables to compute the Kähler potential of the low energy effective $U(1)$ theory. The result is expressed in term of the functions $a(u), a_D(u)$; τ_e is defined as $\tau_e = \frac{da_D}{da}$. The following convention is used for the effective $U(1)$ coupling g_e and θ angle:

$$\tau_e = \frac{\theta}{\pi} + \frac{8\pi i}{g_e^2}. \quad (3)$$

The Kähler metric on the moduli space is given in term of the holomorphic functions $a(u), a_D(u)$:

$$ds^2 = (\text{Im } \tau_e) da d\bar{a} = g du d\bar{u}, \quad g = (\text{Im } \tau_e) a' \bar{a}' = \text{Im } (a'_D \bar{a}'), \quad (4)$$

where $a' = da/du$ and $\bar{a}' = d\bar{a}/d\bar{u}$. Spontaneous supersymmetry breaking in metastable vacua is generated by deforming the theory with a superpotential $\mathcal{W}(u)$. The potential on the moduli space is then

$$V = \frac{|\mathcal{W}'(u)|^2}{g}. \quad (5)$$

In [9] it is shown that by an appropriate choice for the superpotential it is indeed possible to generate a metastable vacuum in almost every point of the moduli space; the proof relies on the fact that any sectional curvature of the Riemann curvature tensor R of the moduli space metric is strictly positive definite in almost every point of the moduli space. This means that (with the exception of a finite number of points in the moduli space) for any two vectors w_1, w_2 on the tangent space,

$$\langle w_1, R(w_2, w_2)w_1 \rangle > 0,$$

for every $w_1, w_2 \neq 0$.

The task of finding metastable vacua is equivalent to finding a local maximum of

$$\frac{1}{V} = \text{Im}(\tau_e) \left| \frac{a'}{\mathcal{W}'} \right|^2. \quad (6)$$

The function $1/V$ is the product of two factors which both don't have local maxima ($\text{Im}(\tau_e)$ because it is an harmonic function; $|a'/\mathcal{W}'|^2$ because it is the squared modulus of an holomorphic function). So the local maximum, when it exists, comes from a non trivial interplay between these two different positive factors.

In order to find metastable vacua one needs to perturb the $\mathcal{N} = 2$ theory with u , u^2 and u^3 operators in the superpotential. We do not know of any example of $\mathcal{N} = 2$ theory where metastable vacua are achieved by just adding the u operator; neither we know about a proof that this can not be achieved.

2.1 How to generate a metastable vacuum on the moduli space

Consider a point on the moduli space u_0 ; then the following parameterization for the superpotential is introduced:

$$\mathcal{W} = \tilde{\mu} W = \tilde{\mu} \left((u - u_0) + \kappa(u - u_0)^2 + \lambda(u - u_0)^3 \right). \quad (7)$$

An explicit expression for the allowed range of κ and λ in order to generate a metastable vacuum in u_0 was found in [12]. In this section this calculation is reviewed and some useful notation is introduced. The potential itself is:

$$V = |\tilde{\mu}|^2 g^{-1}(u, \bar{u}) W'(u) \bar{W}'(\bar{u}), \quad g = (\text{Im}\tau_e(u)) a'(u) \bar{a}'(\bar{u}), \quad (8)$$

The first derivative of the potential is computed for $u = u_0$:

$$\frac{1}{|\tilde{\mu}|^2} \frac{\partial V}{\partial u} = \frac{\partial g^{-1}}{\partial u} W'(u) \bar{W}'(\bar{u}) + g^{-1} W''(u) \bar{W}'(\bar{u}) = \frac{\partial g^{-1}}{\partial u} + 2\kappa g^{-1}. \quad (9)$$

The condition for generating an extremal point at $u = u_0$ is

$$\kappa = -\frac{1}{2} g \frac{\partial g^{-1}}{\partial u}. \quad (10)$$

In order to check if this extremal point is a minimum, one needs to calculate the second derivatives of V :

$$\frac{1}{|\tilde{\mu}|^2} \frac{\partial^2 V}{\partial u^2} = \frac{\partial^2 g^{-1}}{\partial u^2} W' \bar{W}' + 2 \frac{\partial g^{-1}}{\partial u} W'' \bar{W}' + g^{-1} W''' \bar{W}', \quad (11)$$

$$\frac{1}{|\tilde{\mu}|^2} \frac{\partial^2 V}{\partial u \partial \bar{u}} = \frac{\partial^2 g^{-1}}{\partial u \partial \bar{u}} W' \bar{W}' + \frac{\partial g^{-1}}{\partial u} W' \bar{W}'' + \frac{\partial g^{-1}}{\partial \bar{u}} W'' \bar{W}' + g^{-1} W'' \bar{W}''.$$

For $u = u_0$ this reduces to

$$\frac{1}{|\tilde{\mu}|^2} \frac{\partial^2 V}{\partial u^2} = \frac{\partial^2 g^{-1}}{\partial u^2} - 2g \left(\frac{\partial g^{-1}}{\partial u} \right)^2 + 6\lambda g^{-1}, \quad \frac{1}{|\tilde{\mu}|^2} \frac{\partial^2 V}{\partial u \partial \bar{u}} = \frac{\partial^2 g^{-1}}{\partial u \partial \bar{u}} - g \left| \frac{\partial g^{-1}}{\partial u} \right|^2. \quad (12)$$

A minimum is obtained if $\frac{\partial^2 V}{\partial u \partial \bar{u}} > \left| \frac{\partial^2 V}{\partial u^2} \right|$, which gives

$$|\lambda - \lambda_0| < \frac{g}{6} \left(\frac{\partial^2 g^{-1}}{\partial u \partial \bar{u}} - g \left| \frac{\partial g^{-1}}{\partial u} \right|^2 \right) = r_\lambda, \quad (13)$$

where

$$\lambda_0 = \frac{g^2}{3} \left(\frac{\partial g^{-1}}{\partial u} \right)^2 - \frac{g}{6} \frac{\partial^2 g^{-1}}{\partial u^2}. \quad (14)$$

The parameterization in terms of (κ, λ) is useful for the calculation and allows to identify the region of the coupling for which u_0 is metastable: it is a ball with radius r_λ centered in λ_0 in the λ coordinate, and a point in the κ coordinate. But on the other hand it is related to the physical couplings by a non-trivial expression involving u_0 . We find useful to introduce the following parameterization. Dropping an irrelevant constant in (7), and after an appropriate rescaling, the superpotential can be written as

$$\mathcal{W} = \mu (u + \alpha u^2 + \beta u^3), \quad (15)$$

where

$$\alpha = \frac{\kappa - 3\lambda u_0}{1 - 2\kappa u_0 + 3\lambda u_0^2}, \quad \beta = \frac{\lambda}{1 - 2\kappa u_0 + 3\lambda u_0^2}. \quad (16)$$

In the following we will denote as (α_0, β_0) the couplings corresponding to $\lambda = \lambda_0$. The condition to make u_0 metastable is $\lambda = \lambda_0 + r_\lambda \epsilon$, where ϵ is a complex number with $|\epsilon| < 1$. At the first order in $\delta\lambda = \lambda - \lambda_0$, which turns out to be a good approximation for the problem, this translates in

$$\alpha = \alpha_0 + \delta\alpha \epsilon, \quad \delta\alpha = \frac{3(u_0^2 \kappa - u_0) r_\lambda}{(1 - 2\kappa u_0 + 3\lambda u_0^2)^2}, \quad (17)$$

$$\beta = \beta_0 + \delta\beta \epsilon, \quad \delta\beta = \frac{(1 - 2\kappa u_0) r_\lambda}{(1 - 2\kappa u_0 + 3\lambda u_0^2)^2},$$

where the same complex $|\epsilon| < 1$ must be chosen for both (α, β) .

The superpotential (15) also generates some extra supersymmetric vacua at the roots of $\mathcal{W}'(u) = 0$:

$$u_{\pm} = \frac{-\alpha \pm \sqrt{\alpha^2 - 3\beta}}{3\beta}. \quad (18)$$

When the lifetime of the metastable vacuum is considered, also decays to these extra supersymmetric vacua must be taken into account.

3 $\mathcal{N} = 2$ Super Yang-Mills ($N_f = 0$).

In this case the Seiberg-Witten curve [7] is

$$y^2 = (x^2 - \Lambda^4)(x - u). \quad (19)$$

The singularities on the moduli space, which correspond to supersymmetric vacua in the perturbed theory, are at $u_{M,D} = \pm\Lambda^2$. We set for simplicity the dynamical scale Λ to 1.

The functions (a_D, a) can be evaluated by integrating the Seiberg-Witten differential form on the appropriate cycles of the curve [7]. An explicit expression [18] in terms of elliptic integrals is:

$$a(u) = \frac{\sqrt{2(1+u)}}{\pi} E\left(\frac{2}{1+u}\right), \quad (20)$$

$$a_D(u) = \frac{2i}{\pi} \left((1+u) K\left(\frac{1-u}{2}\right) - 2E\left(\frac{1-u}{2}\right) \right).$$

The following conventions (including also $\Pi(\nu, k)$, which will be useful later) are used in this paper:

$$K(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k \sin^2 \phi}}, \quad E(k) = \int_0^{\pi/2} \sqrt{1 - k \sin^2 \phi} d\phi, \quad (21)$$

$$\Pi(\nu, k) = \int_0^{\pi/2} \frac{d\phi}{(1 - \nu \sin^2 \phi) \sqrt{1 - k \sin^2 \phi}}.$$

By adding a superpotential W which is a cubic polynomial in u , one can generate a metastable vacuum at almost every point u_0 of the moduli space, with the exception of the monopole and dyon singularities at $u_{M,D}$. Using the general expressions in section 2.1, the parameters α_0, β_0 can be evaluated as a function of the moduli u_0 .

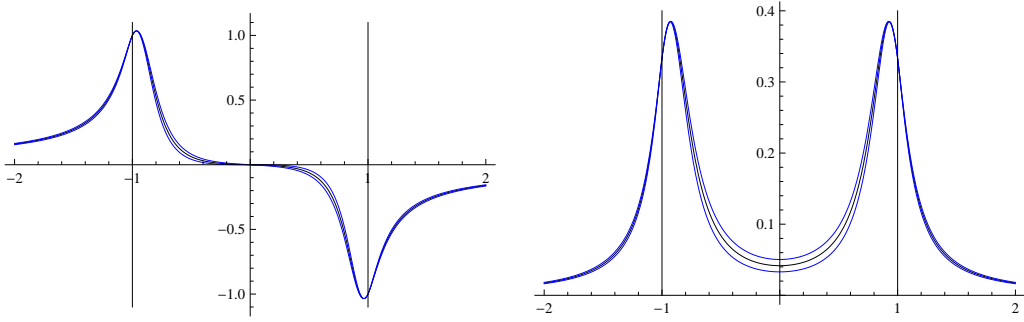


Figure 1: In the black curve, values of $\text{Re } \alpha_0$ (left) and $\text{Re } \beta_0$ (right) are plotted for $N_f = 0$ as a function of $\text{Re } u_0$, for $\text{Im } u_0 = 0$ ($\text{Im } \alpha_0, \text{Im } \beta_0 = 0$). These are the parameters which enter in the superpotential (15). The curves corresponding to $\alpha_0 \pm \delta\alpha$ and $\beta_0 \pm \delta\beta$ are also shown in blue. The vertical lines correspond to the location of the supersymmetric vacua.

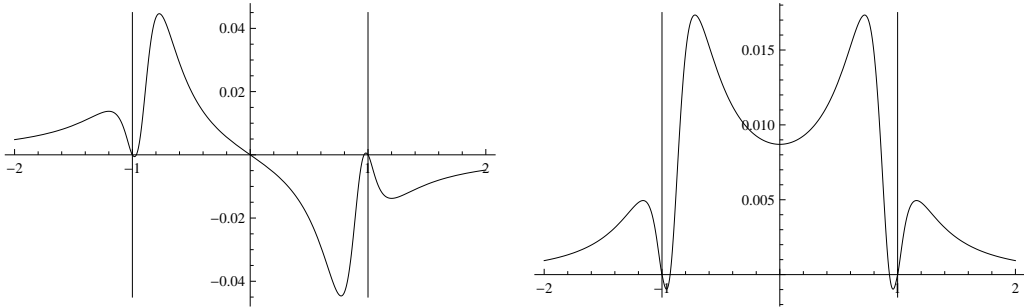


Figure 2: Values of $\delta\alpha$ (left) and $\delta\beta$ (right) for $N_f = 0$, as a function of $\text{Re } u_0$, for $\text{Im } u_0 = 0$. Note that both the functions approach to 0 nearby the supersymmetric vacua at $u_0 = \pm\Lambda$.

The result, for u_0 on the real axis, is shown in figure 1. This agrees with the analytical expressions for $(\kappa, \lambda_0, r_\lambda)$ found in [12]. In agreement with [9, 11], in order to get a metastable vacuum at the origin of the moduli space, we must choose

$$\alpha = 0, \quad \beta \approx 0.0417 \pm 0.0087. \quad (22)$$

Both α_0 and β_0 are regular nearby the supersymmetric minima at $u_{M,D}$. The function $(\delta\alpha, \delta\beta)$, which measure how much we can vary the parameters to keep u_0 metastable, are shown in figure 2. Note that $(\delta\alpha, \delta\beta)$ go both to zero in a linear way as a function of $(u - u_{M,D})$ when approaching the supersymmetric vacua $u_{M,D}$.

The allowed region of parameters in order to generate a metastable vacuum on the real u axis is shown in figure 3. The region where the metastable vacuum is

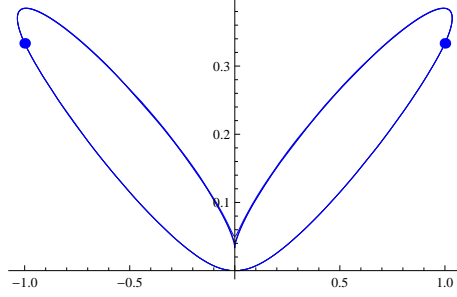


Figure 3: Allowed values of $(\text{Re } \alpha, \text{Re } \beta)$ in order to get a metastable vacuum on the real axis of u_0 ($\text{Im } \alpha, \text{Im } \beta = 0$) for $N_f = 0$. This correspond to a region between two lines that in the scale of the picture are almost coincident. The big dots correspond to the limit $u_0 \rightarrow u_{M,D}$, in which the would be metastable vacuum does not exist because it coincides with a supersymmetric vacuum.

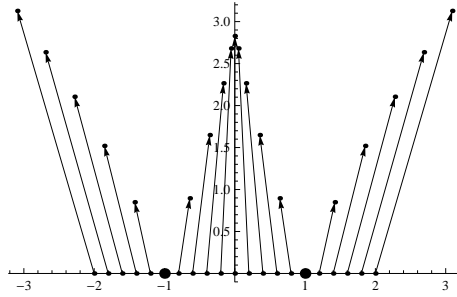


Figure 4: Coordinates of the supersymmetric vacuum $(\text{Re } u_+, \text{Im } u_+)$ as a function of the metastable vacuum u_0 , which we take on the real axis. The big dots correspond to the supersymmetric vacua $u_{M,D}$; in the limit $u_0 \rightarrow u_{M,D}$ also $u_{\pm} \rightarrow u_0$. Each arrow denotes the u_+ vacuum associated with (α_0, β_0) of the particular u_0 .

less fine-tuned is for u_0 nearby the origin, which corresponds to the parameters in Eq. (22).

Using the triangular approximation of [19], the tunneling rate of a metastable vacuum to a supersymmetric vacuum is proportional to e^{-S} , where

$$S \propto \frac{(\Delta u / \Lambda)^4}{\Delta V}, \quad (23)$$

where Δu is the distance between the supersymmetric and the the metastable vacua and ΔV is their difference of potentials. In order to obtain a long-lived metastable vacuum, the tunneling rate must be small. This can be achieved by choosing the over-all constant μ in Eq. (15) enough small (because $\Delta V \propto \mu^2 \Lambda^2$ and Δu is independent

of μ). The tunneling rate must be checked for the decays to all the supersymmetric vacua $u_{susy} = u_{M,D}, u_{\pm}$. The vacua $u_{M,D}$ are independent from the deformation parameters; on the other hand the vacua $u_{+} = u_{-}^{*}$ are a function of (α, β) given by Eq. (18). In figure 4 is it shown u_{+} as a function of u_0 ; the only limit in which u_{+} and u_0 are almost coincident is for $u_0 \approx u_{M,D}$. We conclude that the only regime in which it is problematic to achieve a long-lived metastable vacua is for $u_0 \rightarrow u_{M,D}$, where $u_{M,D}$ are the singularities of the moduli space, which correspond to the supersymmetric vacua which do not depend on (α, β) .

With an appropriate rescaling of Λ , the results of this section for the allowed parameter space of (α, β) apply also to the case with $N_f = 2$ massless fundamentals; this occurs because the structure of the singularities on the moduli space and the Seiberg-Witten curves are the same.

4 $N_f = 4$ and $\mathcal{N} = 2^{*}$ theories

As discussed in the introduction, there are no metastable states in conformal invariant theories; in particular in the $\mathcal{N} = 2$ theory with $N_f = 2N_c$ massless fundamentals and in the theory with $\mathcal{N} = 4$ supersymmetries. This argument is no longer valid in the presence of relevant or irrelevant operators. In this section we will discuss a situation where both the types of operators are present³.

In the conformal invariant cases the effective coupling τ_e is a constant as a function of the moduli. In the $N_c = 2$ case, the functions (a, a_D) are

$$\begin{aligned} a &= \sqrt{\frac{u}{2}}, & a_D &= \tau_e a & \text{for } N_f = 4, \\ a &= \sqrt{2u}, & a_D &= \tau_e a & \text{for } \mathcal{N} = 4. \end{aligned} \tag{24}$$

The moduli space metric is

$$ds^2 = (\text{Im } \tau_e) da d\bar{a} = \frac{1}{8}(u\bar{u})^{-1/2} (\text{Im } \tau_e) du d\bar{u}. \tag{25}$$

From Eqs. (13) we find that $r_{\lambda} = 0$, so it is impossible to stabilize any vacuum with a superpotential of the form (15), which is just a function of u ⁴. Indeed, for

³In [20] it was found that the particularly interesting combinations of relevant operators in the $\mathcal{N} = 4$ theory all carry no anomalous dimensions.

⁴The following simpler proof of this statement was suggested to us by Zohar Komargodski. If τ_e is constant, from Eq. (6) it follows that the potential is proportional to the modulus of a holomorphic function and then no classical metastable state with mass gap can exist. There could be in principle a pseudo-moduli, but it can be checked that it is not the case. This proof can be extended also to the more general case with $N_c > 2$, deformed by $W = \text{Tr } \Phi^k$.

the conformal invariant case the proof of [9] does not apply because the sectional curvature of the moduli space is exactly zero. The situation changes completely when we add finite masses for some of the hypermultiplets; then the proof in [9] applies and it is possible to generate a metastable vacuum at almost every point of the moduli space.

The expression for (a, a_D) can be found by the Seiberg-Witten curve approach [8]. In the case of the massive $N_f = 4$ theory, with two massless squark hypermultiplet ($m_1 = m_2 = 0$) and two massive ones with mass $m_3 = m_4 = m/2$ an explicit expression in terms of elliptic integrals was found in [21]. Due to the fact that the singularity structure is the same, identical expressions apply also for the $\mathcal{N} = 4$ theory deformed by mass term m for the adjoint hypermultiplet ($\mathcal{N} = 2^*$ theory), modulo a trivial rescaling $(a_D, a)_{\mathcal{N}=2^*} = (a_D, 2a)_{N_f=4}$. The underlying physics is rather different; only even instanton corrections contribute in the $N_f = 4$ theory while also odd instantons give a non-zero contribution in the $\mathcal{N} = 2^*$ case.

In the $N_f = 4$ case there are some subtle points in the identification between the UV coupling constant and the IR one which appear in the curve [22]. The coupling τ_e which appear in the SW curve does not correspond with the $SU(2)$ coupling $\tau_{UV} = \theta_{YM}/\pi + 8\pi i/g_{YM}^2$ of the ultraviolet theory. The expression relating these quantities gets contributions from an infinite number of instantons [22]:

$$\tau_e = \tau_{UV} + \frac{i}{\pi} \sum_{n=0,2,4,\dots} a_n q_{UV}^n, \quad (26)$$

where $q_{UV} = \exp(i\pi\tau_{UV})$. For the $\mathcal{N} = 2^*$ theory instead $\tau_e = \tau_{UV} = \theta_{YM}/(2\pi) + 4\pi i/g_{YM}^2$ (note the factor of two in the conventions used for the two different cases).

Other subtleties arise both in the $N_f = 4$ and in the $\mathcal{N} = 2^*$ theories in the identification of the moduli space coordinate z that appears in the curve with the operator $u = \text{Tr } \Phi^2$ of the ultraviolet theory. The expression relating this quantities has the following form [22, 23]:

$$z = u \left(\frac{d\tau}{d\tau_{UV}} \right)^{-1} + R \sum_{n=0,2,4,\dots} \alpha_n q^n, \quad R = \frac{1}{2} \sum_i m_i^2, \quad (27)$$

where different coefficients α_n are needed in the case of $\mathcal{N} = 2^*$ and in the $N_f = 4$ case. This indicates that the operators z^k that will be introduced in the effective description below do not directly correspond to the operators $(\text{Tr } \Phi^2)^k$ in the UV description but that an unknown non-trivial dictionary between the two quantities is needed.

Following [21], we introduce the parameter

$$\tilde{u} = z + \frac{1}{8}m^2 E_1(\tau_e), \quad (28)$$

where E_1 is a function that will be defined in the next paragraph. In this way the parameter \tilde{u} is identified with the physical parameter $u = \text{Tr } \Phi^2$ parameter under the renormalization group flow from the $N_f = 4$ to the $N_f = 2$ theory (or from the $\mathcal{N} = 4$ to the Super-Yang-Mills $N_f = 0$). The variable \tilde{u} then can be identified with u at least in the decoupling limit $m \rightarrow \infty$, $g_{YM} \rightarrow 0$ with $\Lambda \propto m e^{-1/g_{YM}^2}$ fixed.

The Seiberg-Witten curve [8] is:

$$y^2 = 4(x - e_1)(x - e_2)(x - e_3), \quad (29)$$

where

$$\begin{aligned} e_1 &= 0, & e_3 &= (E_2(\tau_e) - E_1(\tau_e))z + \frac{1}{4}m^2(E_2^2(\tau_e) - E_1^2(\tau_e)), \\ e_2 &= (E_3(\tau_e) - E_1(\tau_e))z + \frac{1}{4}m^2(E_3^2(\tau_e) - E_1^2(\tau_e)). \end{aligned} \quad (30)$$

The functions $E_j(\tau_e)$ are defined in term of standard $\theta_{1,2,3}(\tau_e)$ functions:

$$E_1(\tau_e) = \frac{\theta_2^4 + \theta_3^4}{3}, \quad E_2(\tau_e) = -\frac{\theta_1^4 + \theta_3^4}{3}, \quad E_3(\tau_e) = \frac{\theta_1^4 - \theta_2^4}{3}. \quad (31)$$

The following three values of z correspond to singularities of the moduli space metric, where extra massless (electric or magnetic) degrees of freedom are present:

$$z_j = \frac{m^2}{4} E_j(\tau_e), \quad j = 1, 2, 3. \quad (32)$$

These values correspond to supersymmetric vacua; we denote the corresponding values of \tilde{u} as $(\tilde{u}_{s1}, \tilde{u}_{s2}, \tilde{u}_{s3})$.

It is useful to introduce the variables:

$$k^2 = \frac{e_2 - e_3}{e_1 - e_3}, \quad k'^2 = 1 - k^2 = \frac{e_2 - e_1}{e_3 - e_1}. \quad (33)$$

The general strategy is to reduce the computation of a, a_D to the following three elliptic integrals, which can be expressed in term of standard special functions:

$$I_1^j = \oint_{\gamma_j} \frac{dx}{y}, \quad I_2^j = \oint_{\gamma_j} \frac{x dx}{y}, \quad I_3^j(c) = \oint_{\gamma_j} \frac{dx}{(x - c)y}. \quad (34)$$

The explicit expressions [21, 24] are:

$$I_1^1 = \frac{2}{\sqrt{e_1 - e_3}} K(k^2), \quad I_2^1 = \frac{2}{\sqrt{e_1 - e_3}} (e_1 K(k^2) + (e_3 - e_1) E(k^2)), \quad (35)$$

$$I_3^1(c) = \frac{2}{(e_1 - e_3)^{3/2}} \left(\frac{1}{1 - \tilde{c} + k'} K(k^2) + \frac{4k'}{1 + k'} \frac{1}{(1 - \tilde{c})^2 - k'^2} \Pi \left(\nu, \left(\frac{1 - k'}{1 + k'} \right)^2 \right) \right)$$

where

$$\tilde{c} = \frac{c - e_3}{e_1 - e_3}, \quad \nu = \left(\frac{1 - \tilde{c} + k'}{1 - \tilde{c} - k'} \right)^2 \left(\frac{1 - k'}{1 + k'} \right)^2,$$

and I_j^2 can be obtained from I_j^1 by exchanging e_1 and e_3 (this exchanges k and k'). The functions K, E, Π are standard elliptic integrals, defined with the conventions in Eq. (21).

Then the explicit expression for (a, a_D) can be evaluated, by integrating the SW differential; the result is

$$a(z) = \frac{\sqrt{2}}{\pi} (z - z_1) \left(I_1^1 - \frac{m^2}{4} \theta_2^4 \theta_3^4 I_3^1 \left(\frac{m^2}{4} \theta_2^4 \theta_3^4 \right) \right), \quad (36)$$

$$a_D(z) = \frac{\sqrt{2}}{\pi} (z - z_1) \left(I_1^2 - \frac{m^2}{4} \theta_2^4 \theta_3^4 I_3^2 \left(\frac{m^2}{4} \theta_2^4 \theta_3^4 \right) \right).$$

The value of the low energy coupling τ_e is

$$\tau_e = \frac{da_D}{da} = \frac{I_1^2}{I_1^1}. \quad (37)$$

Using the general expression in section 2.1, we can now study the allowed region of the parameters $(\tilde{\alpha}, \tilde{\beta})$ in

$$\mathcal{W} = \mu(\tilde{u} + \tilde{\alpha}\tilde{u}^2 + \tilde{\beta}\tilde{u}^3)$$

in order to generate a metastable vacuum in \tilde{u}_0 . The variable \tilde{u} can be identified with $u = \text{Tr } \Phi^2$ just in a decoupling limit where we recover the massless $N_f = 2$ case for the massive $N_f = 4$ theory and the $N_f = 0$ case for the $\mathcal{N} = 2^*$ theory. The full expression for \tilde{u} as a function of u is unknown in both the cases. Plots for the quantities $(\tilde{\alpha}_0, \tilde{\beta}_0, \delta\tilde{\alpha}, \delta\tilde{\beta})$, for the value $\tau = 1.1i$, are shown in figure 5, 6. The allowed region of parameters in order to generate a metastable vacuum on the real \tilde{u}_0 axis is shown in figure 7.

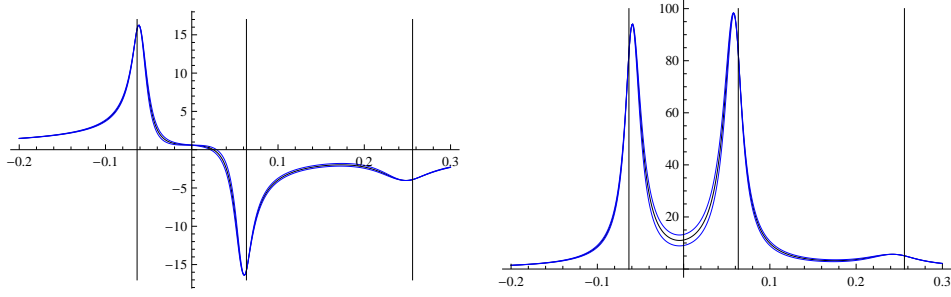


Figure 5: Left: $\text{Re } \tilde{\alpha}_0$, Right: $\text{Re } \tilde{\beta}_0$ for \tilde{u}_0 real ($\text{Im } \tilde{\alpha}_0, \text{Im } \tilde{\beta}_0 = 0$) for $N_f = 4$ and $\tau_e = 1.1i$ (the units are set by $m = 1$). The same plot applies to the $\mathcal{N} = 2^*$ theory.

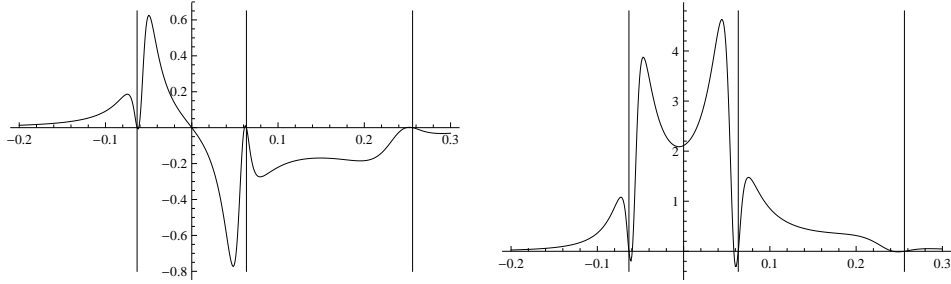


Figure 6: Left: $\delta \tilde{\alpha}$, Right: $\delta \tilde{\beta}$ for \tilde{u}_0 real ($N_f = 4$, $\tau_e = 1.1i$ and $m = 1$).

The tunneling rate can be estimated by Eq. (23). In order to check that the metastable vacuum is long-lived, we must check the decays to all the supersymmetric vacua $\tilde{u}_{susy} = \tilde{u}_{s1}, \tilde{u}_{s2}, \tilde{u}_{s3}, \tilde{u}_{\pm}$. In figure 8 the supersymmetric vacuum \tilde{u}_+ is shown as a function of \tilde{u}_0 ; \tilde{u}_+ and \tilde{u}_0 are almost coincident just for $\tilde{u}_0 \rightarrow \tilde{u}_{s1}, \tilde{u}_{s2}, \tilde{u}_{s3}$. The only limit in which it is problematic to achieve a long-lived metastable vacua is when \tilde{u}_0 is chosen very nearby to these values.

It is interesting that in the case with fundamental hypermultiplets it is possible to obtain metastable vacua also in another limit. Let us start with $\mathcal{N} = 2$ $SU(N_c)$ gauge theory with $N_f = 2N_c$ fundamentals. In $\mathcal{N} = 1$ language, the field content is given by a vector superfield, and adjoint chiral superfield Φ , and N_f fundamentals and anti-fundamentals Q and \tilde{Q} . The superpotential reads:

$$W = \sum_{i=1 \dots N_f} \tilde{Q}_i \Phi Q_i. \quad (38)$$

Let us then fix an integer \tilde{N}_f with $N_c < \tilde{N}_f < 3/2 N_c$. The following mass terms are

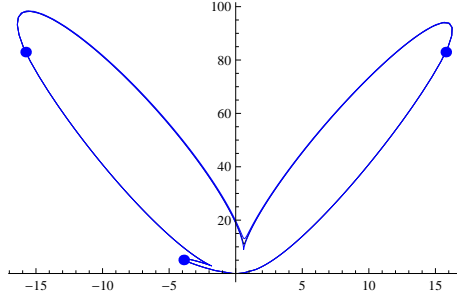


Figure 7: Allowed values of $(\text{Re } \tilde{\alpha}, \text{Re } \tilde{\beta})$ in order to get a metastable vacuum on the real axis of \tilde{u}_0 (with $\text{Im } \tilde{\alpha}, \text{Im } \tilde{\beta} = 0$), for $N_f = 4$, $\tau_e = 1.1i$, $m = 1$.

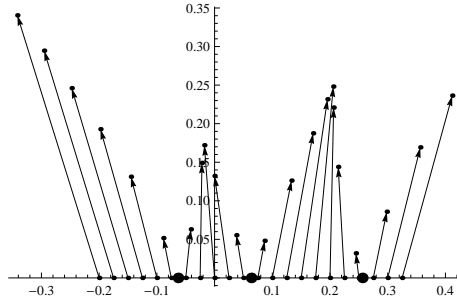


Figure 8: Coordinates of the supersymmetric vacuum $(\text{Re } \tilde{u}_+, \text{Im } \tilde{u}_+)$ as a function the metastable vacuum \tilde{u}_0 , for $N_f = 4$, $\tau_e = 1.1i$, $m = 1$.

then introduced in the superpotential:

$$\Delta W = M\Phi^2 + \sum_{i=\tilde{N}_f+1\dots N_f} MQ\tilde{Q} + \sum_{i=1\dots\tilde{N}_f} mQ\tilde{Q}. \quad (39)$$

Then we consider the limit $g_{YM} \rightarrow 0$. In this limit, at the scale M some of the fields of the theory decouple and do not contribute any more to the β function for the gauge coupling. In the far infrared, the theory reduces to $\mathcal{N} = 1$ SQCD with dynamical scale $\Lambda \approx Me^{-1/g_{YM}^2}$. The mass term m is then chosen in such a way that $m \ll \Lambda$. We can now embed the ISS model [1] in the infrared of the theory. The range $N_c < \tilde{N}_f < 3/2N_c$ is needed in order for the Seiberg dual to be in the free magnetic phase. The mass term m is also needed in order to have metastable supersymmetry breaking. This metastable vacuum is rather different from the ones that are found on the Coulomb branch in the limit of small perturbation from the

$\mathcal{N} = 2$ limit. It is not known if these two kinds of vacua can be related by a continuous change of the parameters.

5 An example with a conformal point: the $N_f = 2$ theory

In this section we discuss the case of $N_f = 2$ massive fundamentals, which is interesting because for a critical value of the hypermultiplet mass ($m_1 = m_2 = \Lambda/2$) an infrared Argyres-Douglas fixed point [16] exists.

The Seiberg-Witten curve [8] in this case is:

$$y^2 = x^2(x - u) - \frac{\Lambda^4}{64}(x - u) + \frac{\Lambda^2}{4}m_1m_2x - \frac{\Lambda^4}{64}(m_1^2 + m_2^2). \quad (40)$$

In this section we set $m_1 = m_2 = m$. The singular points of the moduli space of vacua are at

$$u_{s1} = -\frac{\Lambda^2}{8} - \Lambda m, \quad u_{s2} = -\frac{\Lambda^2}{8} + \Lambda m, \quad u_{s3} = m^2 + \frac{\Lambda^2}{8}. \quad (41)$$

These values correspond to supersymmetric vacua.

The roots of the polynomial which defines the cubic are

$$e_1 = \frac{u}{6} - \frac{\Lambda^2}{16} + \frac{1}{2}\sqrt{u + \frac{\Lambda^2}{8} + \Lambda m}\sqrt{u + \frac{\Lambda^2}{8} - \Lambda m}, \quad (42)$$

$$e_2 = -\frac{u}{3} + \frac{\Lambda^2}{8},$$

$$e_3 = \frac{u}{6} - \frac{\Lambda^2}{16} - \frac{1}{2}\sqrt{u + \frac{\Lambda^2}{8} + \Lambda m}\sqrt{u + \frac{\Lambda^2}{8} - \Lambda m},$$

where a translation in such a way that $\sum_i e_i = 0$ is done for convenience. We can then define the expressions for I_i^j in the same way as for $N_f = 4$, using the new e_j given in Eq. (42) and k, k' defined as in Eq. (33) in term of the new e_j . These expressions are now function of the moduli space coordinate u instead that of z as in the $N_f = 4$ case.

An explicit expression for (a, a_D) was computed in [24]:

$$a = \frac{\sqrt{2}}{4\pi} \left(\frac{4}{3} u I_1^1 - 2 I_2^1 - \frac{\Lambda^2}{2} m^2 I_3^1 \left(-\frac{\Lambda^2}{8} - \frac{u}{3} \right) \right) + \frac{m}{\sqrt{2}}, \quad (43)$$

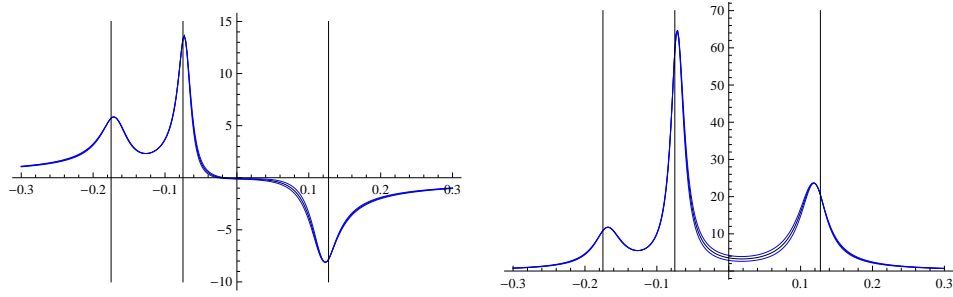


Figure 9: Left: α_0 , Right: β_0 on the real u_0 axis for $N_f = 2$ and $m = 0.05\Lambda$. The units are fixed by $\Lambda = 1$.

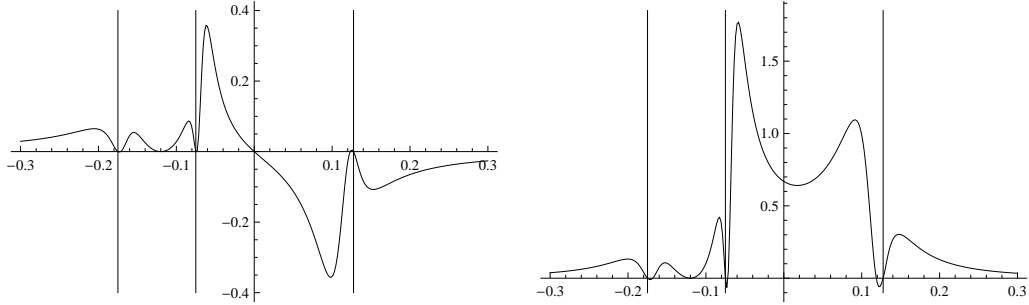


Figure 10: Values of $\delta\alpha$ (left) and $\delta\beta$ (right) for $N_f = 2$ for $m = 0.05\Lambda$ on the real u_0 axis.

$$a_D = \frac{\sqrt{2}}{4\pi} \left(\frac{4}{3} u I_1^2 - 2 I_2^2 - \frac{\Lambda^2}{2} m^2 I_3^2 \left(-\frac{\Lambda^2}{8} - \frac{u}{3} \right) \right).$$

Using the general expressions in section 2.1, we can then compute the allowed region of the parameters (α, β) . The values of $(\alpha_0, \beta_0, \delta\alpha, \delta\beta)$ in the case of $m = 0.05\Lambda$ on the real u_0 axis are shown in figures 9 and 10. Similar plots for the critical mass $m_c = 0.5\Lambda$ are shown in figure 11 and 12; the allowed region of parameters for the two masses choices is shown in figure 13.

For generic m there are three singularities on the moduli space (see Eq. (41)); for $m_c = \Lambda/2$, two of these singularities collide and an Argyres-Douglas point appears [16], corresponding to a non-trivial interacting conformal fixed point in the IR. The AD fixed point is at $u_{AD} = 0.375\Lambda^2$. For $m > m_c$ one of the singularities is at weak coupling and corresponds to massless electric degrees of freedoms; the other two singularities are in the strong coupling region and correspond to the massless monopole and dyon points of the $N_f = 0$ case. For $m < m_c$ all the singularities are

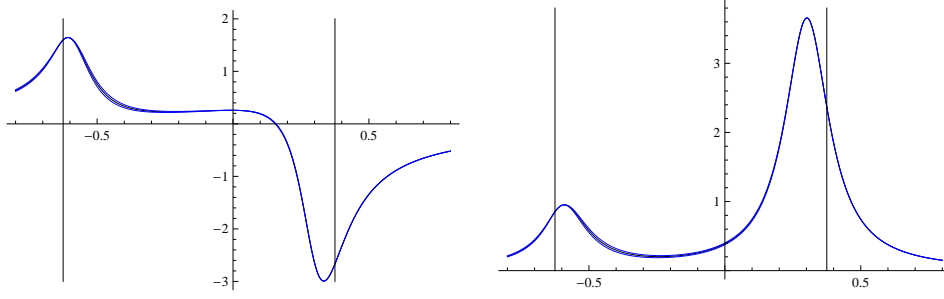


Figure 11: Left: α_0 , Right: β_0 on the real u_0 axis for $N_f = 2$ and $m = m_c = 0.5\Lambda$ (for this value there is an Argyres-Douglas point). The AD fixed point is at $u_{AD} = 0.375\Lambda^2$.

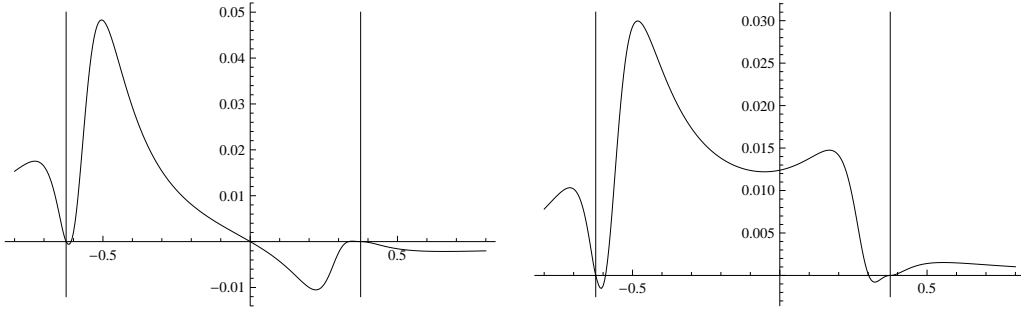


Figure 12: Values of $\delta\alpha$ (left) and $\delta\beta$ (right) for $N_f = 2$ for $m = m_c$, on the real u_0 axis.

at strong coupling.

The tunneling rate can be estimated by Eq. (23). The decays to all the supersymmetric vacua $u_{susy} = u_{s1}, u_{s2}, u_{s3}, u_{\pm}$ must be checked to achieve a long-lived vacuum. In figure 14 u_+ is shown as a function of u_0 ; u_+ and u_0 are almost coincident just for $u_0 \rightarrow u_{s1}, u_{s2}, u_{s3}$. This is the only limit in which it is problematic to achieve a long-lived metastable vacua.

It is possible to generate a local minimum nearby the Argyres-Douglas point, but the allowed $(\delta\alpha, \delta\beta)$ is rather small nearby this point. In the numerical example that we considered, we obtain that $\delta\alpha, \delta\beta \propto (u_0 - u_{AD})^3$. Nearby a non-conformal supersymmetric vacuum instead we obtain that $\delta\alpha, \delta\beta \propto (u_0 - u_{susy})$. Of course in these limits the parameter μ must be very small in order to assure a long life to the metastable vacua.

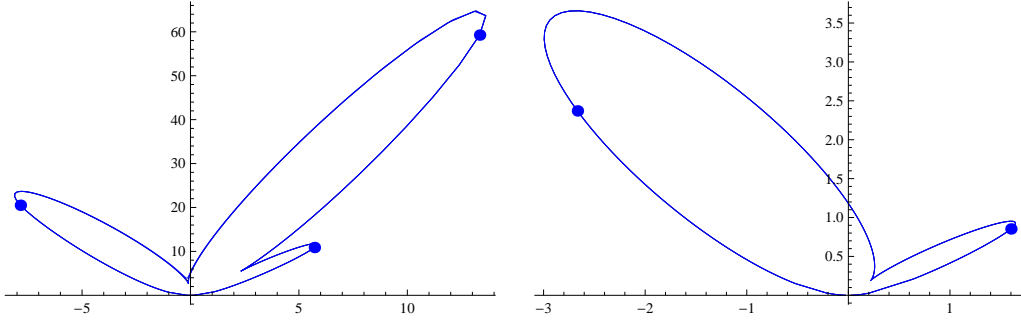


Figure 13: Allowed values of $(\text{Re } \alpha, \text{Re } \beta)$ in order to get a metastable vacuum on the real axis of u_0 (with $\text{Im } \alpha, \text{Im } \beta = 0$) for $N_f = 2$, with $m = 0.05\Lambda$ (left) and $m = m_c = 0.5\Lambda$ (right).

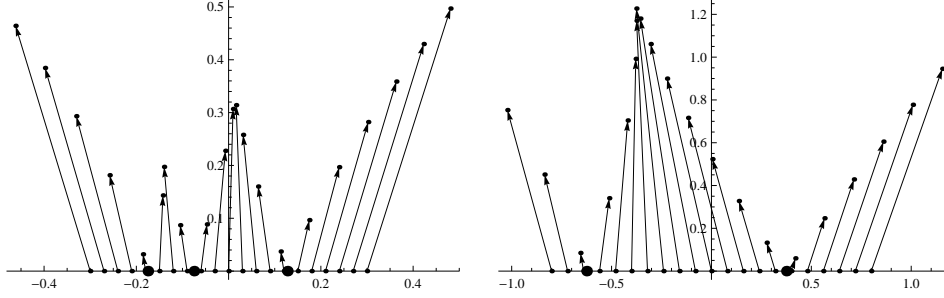


Figure 14: Coordinates of the supersymmetric vacuum $(\text{Re } u_+, \text{Im } u_+)$ as a function of the metastable vacuum u_0 for $N_f = 2$, with $m = 0.05\Lambda$ (left) and $m = m_c = 0.5\Lambda$ (right).

6 Comments on direct gauge mediation

Direct gauge mediation is a well studied topic in the framework of models which exhibit spontaneous symmetry breaking (see for example [17, 3] for reviews). Many models of direct gauge mediation based on generalizations of the O’Raifeartaigh model have anomalously light gauginos in comparison to the sfermions, even in the absence of an R-symmetry; as discussed in [25], the underlying reason for which gaugino masses vanish at the leading-order in SUSY-breaking is due to the fact that no unstable region exists in the pseudo-moduli space. Phenomenologically gauginos whose mass scale is lighter than the electroweak scale are very likely ruled out; thus sfermions need to be made rather heavy; this has its own aesthetic problems because heavy sfermions induce a large correction to the Higgs mass and reintroduce the hierarchy issue. This is a feature also of direct gauge mediation from the ISS model

[1], because in this case the metastable vacuum is absolutely stable in the effective low energy description (supersymmetry is restored just due to non-perturbative effects). A possible way to avoid light gauginos is to consider uplifted vacua [26], which are vacua of even higher energy compared to the lowest supersymmetry breaking vacuum; in their presence an unstable region in pseudo-moduli space becomes allowed. It is interesting that in the class of perturbed $\mathcal{N} = 2$ theories discussed in this paper it is also possible to obtain gaugino masses at the leading order in SUSY-breaking; this is not in contradiction with [25], because the metastable vacua that we consider are not absolutely stable in any low energy approximation (see also [27] for a discussion).

In the weakly coupled region the fields Q, \tilde{Q} can be identified with the messengers. For $N_c = 2$ the so called spurion of supersymmetry breaking corresponds to the adjoint field $\Phi = a(u_0) \sigma_3$, where σ_3 is a Pauli matrix. If the squark masses are set to zero, ordinary gauge mediation (OGM) is realized. If the squark masses are not zero, the gauge mediation mechanism is in the more general class studied in [28]; the gaugino and the sfermion masses are:

$$m_\lambda = \frac{\alpha_r}{4\pi} \Lambda_G, \quad m_{\tilde{f}}^2 = 2C_{\tilde{f}} \left(\frac{\alpha_r}{4\pi} \right)^2 \Lambda_S^2, \quad (44)$$

where

$$\Lambda_G = F^a (\partial_a (\log \det \mathcal{M})) , \quad \Lambda_S^2 = \frac{1}{2} |F^a|^2 \frac{\partial^2}{\partial a \partial \bar{a}} \sum_i (\log |\mathcal{M}_i|^2)^2 , \quad (45)$$

and $\alpha_r = g_F^2/(4\pi)$, where g_F is the gauge coupling at the messenger scale and $C_{\tilde{f}}$ is the quadratic Casimir of the representation of the sfermion \tilde{f} .

Consider for example the case with $N_c = N_f = 2$ and with two identical masses for the hypermultiplets $m_1 = m_2 = m$. For $m = 0$ the theory has an enhanced $SO(4)$ flavor global symmetry; for $m \neq 0$ this symmetry is broken to $SU(2)_F \times U(1)_F$, where the $U(1)_F$ corresponds to a squark number. The BPS mass formula for a state with electric and magnetic charge (n_e, n_m) and with $U(1)_F$ charge s is:

$$M_{BPS} = |\sqrt{2}n_m a_D - \sqrt{2}n_e a + s m|. \quad (46)$$

The $U(1)_F$ symmetry is gauged and coupled to a an external sector, with coupling constant α_r . The matrix $\mathcal{M} = \sqrt{2} a(u_0) \sigma_3 + m$ is the messenger mass matrix. A direct evaluation gives:

$$\Lambda_G = F^a \frac{4a}{2a^2 - m^2}, \quad \Lambda_S^2 = |F^a|^2 \frac{4(2|a|^2 + |m|^2)}{|2a^2 - m^2|^2}. \quad (47)$$

In the $m \rightarrow 0$ limit, OGM is recovered; the effective number of messengers N_{eff} is

$$N_{eff} = \frac{\Lambda_G^2}{\Lambda_S^2} = 2. \quad (48)$$

This shows that the gauginos are not anomalously light in comparison to the sfermions for the metastable vacua in the weakly coupled regime.

In the strongly coupled region of the moduli space the fields Q, \tilde{Q} can not be identified any more with the messengers; in this regime monopoles and dyons which carry flavor quantum numbers become lighter than the squarks, which also can become unstable particles due to the crossing of a curve of marginal stability. The calculation of the gauge mediation masses in principle requires the calculation of the current-current correlators of the global symmetries, in the formalism introduced in [29]. An explicit expression for the gaugino masses at the leading order in SUSY-breaking in perturbed $\mathcal{N} = 2$ theories was found in [30], for generic N_f and N_c . In this more general case the Coulomb branch can be parameterized by N_c eigenvalues a_k , with the constraint $\sum a_k = 0$. In order to facilitate the computation of the gaugino masses, in [30] the global symmetry $U(1)_F \times SU(N_f)$ is gauged by introducing a full $\mathcal{N} = 2$ vector hypermultiplet; in this way the mass parameters m_a of the original $SU(N_c)$ theory get identified with the adjoint scalar eigenvalues of the $\mathcal{N} = 2$ $U(1)_F \times SU(N_f)$ vector hypermultiplet. In the limit in which the spectator $U(1)_F \times SU(N_f)$ gauge couplings are small, the gaugino mass matrix is

$$(m_\lambda^{GGM})_{ab} = g_F^2 \frac{i}{8\pi} (F^i \mathcal{F}_{iab} - (\mathcal{F}_{ijm} F^m)^{-1} \mathcal{F}_{aik} \mathcal{F}_{bjl} F^k F^l), \quad (49)$$

where $\mathcal{F}(a_k, m_a)$ is the prepotential and F^k is the F-term of the field a_k . Subscripts under \mathcal{F} denote differentiations; the indices i, j, k, l, m correspond to the eigenvalues Φ^i of the adjoint field, while the indices a, b correspond to the mass matrix eigenvalues m_a .

We can then apply Eq. (49) to the gauge mediation of the $U(1)_F$ symmetry of the $N_f = N_c = 2$ theory; the mass of the gaugino then is

$$m_\lambda^{GGM} = g_F^2 |F^a \mathcal{A}|, \quad \mathcal{A} = \frac{i}{8\pi} \left(\frac{\partial^2 a_D}{\partial m^2} - \left(\frac{\partial \tau_e}{\partial m} \right)^2 \frac{\partial a}{\partial \tau_e} \right), \quad (50)$$

where F^a is the F-term for the field a :

$$F^a = \frac{1}{\text{Im } \tau_e} \frac{d\bar{W}}{d\bar{u}} \frac{d\bar{u}}{d\bar{a}}. \quad (51)$$

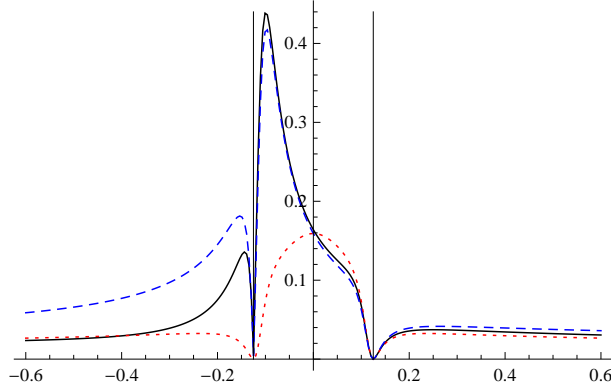


Figure 15: The gaugino mass m_λ^{GGM} , as computed from the general gauge mediation expression Eq. (50), is shown in the solid line in units of $g_F^2 \mu$ as a function of u_0 . The mass m_λ^{W} as computed from Eq. (52) is shown in the dotted line; this is a good approximation in the semiclassical region of the moduli space, for $u_0 \rightarrow \pm\infty$. The mass m_λ^{D} as computed from Eq. (53) is shown in the dashed line; this gives a good approximation inside the marginal stability curve ($u_D < u_0 < u_M$). The location of u_D and u_M is shown by the vertical lines.

In the following we will restrict to the case $m = 0$. In this case the structure of the singularities is identical to the $N_f = 0$ case [8]. There is a dyon singularity at $u_D = -\Lambda^2/8$; in correspondence of this vacuum two dyon states (which are $SU(2)_F$ singlets) with electric and magnetic charges $(n_e, n_m) = (1, -1)$ and with $U(1)_F$ charge ± 1 become massless. There is a monopole singularity at $u_M = \Lambda^2/8$; for this value an $SU(2)_F$ doublet of monopoles with $n_m = 1$ and with zero $U(1)_F$ charge becomes massless.

It is interesting to compare the exact expression in Eq. (50) with the semiclassical formula Eq. (47), which takes into account just the contribution of the Q, \tilde{Q} messengers:

$$m_\lambda^{\text{W}} = \frac{g_F^2}{8\pi^2} \left| \frac{F^a}{a} \right|. \quad (52)$$

In the neighborhood of the moduli space singularity at $u_D = -\Lambda^2/8$, another approximation can be used; nearby this singularity, two dyons (with $(n_e, n_m) = (1, -1)$ and with global $U(1)_F$ charge ± 1) become almost massless. In this limit these dyons give the dominant contribution to the gauge mediation masses; the expressions (44, 45) can be used, by integrating in the messengers in the form of dyon superfields D, \tilde{D} ⁵, which couple to the adjoint field with the superpotential

⁵ A canonical Kähler potential is used for the dyonic fields D, \tilde{D} . This is justified nearby the

$\tilde{W} = \sqrt{2}(a + a_D)D\tilde{D}$. The expression for the gaugino mass in this limit then is:

$$m_\lambda^D = \frac{g_F^2}{8\pi^2} \left| F^a \frac{1 + \tau_e}{a + a_D} \right|. \quad (53)$$

The result of a numerical calculation for the theory with $m = 0$ is shown in figure 15. For each point of the moduli space, the coefficients (α_0, β_0) are computed; these coefficients specify the superpotential W used for each point in the moduli space. Then the gaugino mass is calculated using Eq. (50), see the solid line in figure 15. In the weakly coupled region of the moduli space $u_0 \rightarrow \pm\infty$ the approximation Eq. (52) can be used (see the dotted line in figure 15). The approximation in Eq. (53) is plotted in the dashed curve; this gives a good approximation in the region $u_D < u_0 < u_M$.

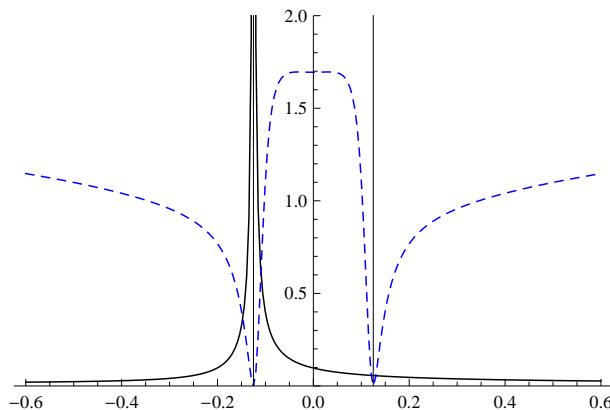


Figure 16: In the solid line, the absolute value of the coefficient \mathcal{A} defined in Eq. (50) is shown as a function of u_0 in units of $1/\Lambda$; it diverges for $u_D = -\Lambda^2/8$, where a dyon charged under $U(1)_F$ becomes massless. In the dashed line, the absolute value of the F-term F^a is shown in units of $\mu\Lambda$; it is zero in correspondence of the supersymmetric vacua $u_{M,D} = \pm\Lambda^2/8$. From this figure we can check that in correspondence of u_D there is a massless particle charged under $U(1)_F$.

The gaugino mass is proportional to the product between the F-term and the expression \mathcal{A} , as defined in defined in Eq. (50). The F-term F^a tends to zero as u approaches the value of the supersymmetric vacua $u_{M,D} = \pm\Lambda^2/8$ (see figure 16).

moduli space singularity u_D , because in this region the dual gauge coupling is weak. In general, there could be terms in the Kähler potential which mix D, \tilde{D} with u . These terms (whose form is much restricted by $\mathcal{N} = 2$ supersymmetry) could in principle induce a non-zero messenger supertrace and make the calculation of the soft masses sensitive to physics at scales higher than the messenger scale [31].

The coefficient \mathcal{A} tends to infinity for $u_D = -\Lambda^2/8$; this is due to the fact that in this vacuum some particles charged under the global $U(1)_F$ symmetry become massless.

As shown in figure 15, Eq. (53) gives a rather good approximation for the gaugino masses in all the strong coupling region with $u_D < u_0 < u_M$. The reason for which this formula works so well in this region of the moduli space is probably due to the fact that we are inside of the marginal stability curve; in this region the only stable BPS states are the monopole (which is uncharged under $U(1)_F$) and the $(1, -1)$ dyons [32]. The relevant gauge mediation physics is captured by the contribution of the dyons, evaluated as at weak coupling. It is then natural to use this approximation also for the sfermions masses, which are proportional to

$$\Lambda_S^2 = 2|F^a|^2 \frac{|1 + \tau_e|^2}{|a + a_D|^2} = \frac{\Lambda_G^2}{2}. \quad (54)$$

This is the same result as in the weakly coupled regime in Eq (48); this suggests that the ordinary gauge mediation relation $N_{eff} = 2$ is satisfied with good approximation also in the strong coupling region $u_D < u_0 < u_M$. The gaugino and the sfermions masses are then of comparable order also in this regime.

7 Conclusions

In this note we studied the issue of metastable vacua in $\mathcal{N} = 2$ theories perturbed by the superpotential in Eq. (1). The allowed region of the parameters (α, β) in order to obtain a metastable vacuum on the Coulomb branch was determined in some examples with $N_c = 2$ and different matter content. A general feature in the asymptotically free cases is that the parameters must be considerably fine-tuned for large $u_0 \gg \Lambda$, while in the strongly coupled region a smaller degree of fine tuning is needed. Another feature is that it is more difficult to generate metastable vacua in the conformal setting; in order to achieve this in the $N_f = 4$ and in the $\mathcal{N} = 4$ theories the conformal symmetry must be explicitly broken also by a mass term for some of the hypermultiplets. Also, we find that it is hard to achieve a metastable vacuum nearby an infrared Argyres-Douglas conformal point; in the explicit example that we studied we found that $\delta\alpha, \delta\beta$ vanish as $(u_0 - u_{AD})^3$, which is stronger than nearby the other supersymmetric vacua, where we find that $\delta\alpha, \delta\beta$ vanish in a linear way in $(u_0 - u_{susy})$.

Direct gauge mediation can be implemented with sizable gaugino masses already at the leading order in SUSY breaking. In particular, in the case of zero hypermultiplets masses, ordinary gauge mediation is realized.

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Appendix. The weakly coupled limit

For $N_f = 0 \dots 3$, in the weakly coupled region of the moduli space, $u \gg \Lambda^2, m_i^2$, the form of (a_D, a) is determined by the one-loop β function, which (setting $\Lambda = 1$) leads to

$$a(u) \approx \sqrt{\frac{u}{2}}, \quad a_D(u) \approx \frac{i}{4\pi}(4 - N_f)\sqrt{2u} \log u. \quad (55)$$

The moduli space metric is

$$g = \frac{4 - N_f}{16\pi} \frac{2 + \log |u|}{|u|}. \quad (56)$$

We can then write a compact expression the four functions $(\alpha_0, \beta_0, \delta\alpha, \delta\beta)$:

$$\begin{aligned} \alpha_0 &= -\frac{8 + 5 \log |u_0|^2}{u_0(44 + 15 \log |u_0|^2)} \approx -\frac{1}{3u_0}, \\ \beta_0 &= \frac{4 + 3 \log |u_0|^2}{3u_0^2(44 + 15 \log |u_0|^2)} \approx \frac{1}{15u_0^2}, \\ \delta\alpha &= -\frac{8(18 + 5 \log |u_0|^2)}{\bar{u}_0(4 + \log |u_0|^2)(44 + 15 \log |u_0|^2)^2} \approx -\frac{8}{45\bar{u}_0(\log |u_0|^2)^2}, \\ \delta\beta &= \frac{16(10 + 3 \log |u_0|^2)}{3|u_0|^2(4 + \log |u_0|^2)(44 + 15 \log |u_0|^2)^2} \approx \frac{16}{225|u_0|^2(\log |u_0|^2)^2}. \end{aligned} \quad (57)$$

A general feature of the weakly-coupled region is that the perturbation parameters (α, β) must be small and rather fine tuned (with $\beta \approx 3/5\alpha^2$) in order to achieve metastability.

References

- [1] K. A. Intriligator, N. Seiberg and D. Shih, JHEP **0604** (2006) 021 [arXiv:hep-th/0602239].
- [2] K. A. Intriligator and N. Seiberg, Class. Quant. Grav. **24** (2007) S741 [arXiv:hep-ph/0702069].
- [3] R. Kitano, H. Ooguri and Y. Ookouchi, arXiv:1001.4535 [hep-th].
- [4] N. Seiberg, Nucl. Phys. B **435**, 129 (1995) [arXiv:hep-th/9411149].
- [5] K. I. Izawa, F. Takahashi, T. T. Yanagida and K. Yonekura, Phys. Rev. D **80**, 085017 (2009) [arXiv:0905.1764 [hep-th]]; T. T. Yanagida and K. Yonekura, arXiv:1002.4093 [hep-th].
- [6] A. Amariti, L. Girardello, A. Mariotti and M. Siani, arXiv:1003.0523 [hep-th].
- [7] N. Seiberg and E. Witten, Nucl. Phys. B **426** (1994) 19 [Erratum-ibid. B **430** (1994) 485] [arXiv:hep-th/9407087].
- [8] N. Seiberg and E. Witten, Nucl. Phys. B **431** (1994) 484 [arXiv:hep-th/9408099].
- [9] H. Ooguri, Y. Ookouchi and C. S. Park, Adv. Theor. Math. Phys. **12** (2008) 405 [arXiv:0704.3613 [hep-th]].
- [10] J. Marsano, H. Ooguri, Y. Ookouchi and C. S. Park, Nucl. Phys. B **798** (2008) 17 [arXiv:0712.3305 [hep-th]].
- [11] G. Pastras, arXiv:0705.0505 [hep-th].
- [12] E. Katifori and G. Pastras, arXiv:0811.3393 [hep-th].
- [13] M. Arai, C. Montonen, N. Okada and S. Sasaki, Phys. Rev. D **76** (2007) 125009 [arXiv:0708.0668 [hep-th]]. M. Arai, C. Montonen, N. Okada and S. Sasaki, JHEP **0803** (2008) 004 [arXiv:0712.4252 [hep-th]].
- [14] L. Mazzucato, Y. Oz and S. Yankielowicz, JHEP **0711**, 094 (2007) [arXiv:0709.2491 [hep-th]]; J. Marsano, K. Papadodimas and M. Shigemori, Nucl. Phys. B **804**, 19 (2008) [arXiv:0801.2154 [hep-th]]; L. Hollands, J. Marsano, K. Papadodimas and M. Shigemori, JHEP **0810**, 102 (2008) [arXiv:0804.4006 [hep-th]].

- [15] E. Rabinovici, B. Saering and W. A. Bardeen, Phys. Rev. D **36**, 562 (1987);
D. J. Amit and E. Rabinovici, Nucl. Phys. B **257**, 371 (1985); D. S. Berman
and E. Rabinovici, arXiv:hep-th/0210044.
- [16] P. C. Argyres and M. R. Douglas, Nucl. Phys. B **448** (1995) 93
[arXiv:hep-th/9505062]; P. C. Argyres, M. Ronen Plesser, N. Seiberg and
E. Witten, Nucl. Phys. B **461** (1996) 71 [arXiv:hep-th/9511154].
- [17] G. F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999)
[arXiv:hep-ph/9801271].
- [18] A. Bilal, arXiv:hep-th/9601007.
- [19] M. J. Duncan and L. G. Jensen, Phys. Lett. B **291**, 109 (1992).
- [20] M. B. Einhorn, G. Goldberg and E. Rabinovici, Nucl. Phys. B **256**, 499 (1985).
- [21] F. Ferrari, Nucl. Phys. B **501** (1997) 53 [arXiv:hep-th/9702166].
- [22] N. Dorey, V. V. Khoze and M. P. Mattis, Nucl. Phys. B **492** (1997) 607
[arXiv:hep-th/9611016].
- [23] N. Dorey, V. V. Khoze and M. P. Mattis, Phys. Lett. B **396** (1997) 141
[arXiv:hep-th/9612231].
- [24] A. Bilal and F. Ferrari, Nucl. Phys. B **516** (1998) 175 [arXiv:hep-th/9706145].
- [25] Z. Komargodski and D. Shih, JHEP **0904** (2009) 093 [arXiv:0902.0030 [hep-
th]].
- [26] R. Kitano, H. Ooguri and Y. Ookouchi, Phys. Rev. D **75** (2007) 045022
[arXiv:hep-ph/0612139]; A. Giveon, A. Katz and Z. Komargodski, JHEP **0907**
(2009) 099 [arXiv:0905.3387 [hep-th]]; S. A. Abel, J. Jaeckel and V. V. Khoze,
Phys. Lett. B **682** (2010) 441 [arXiv:0907.0658 [hep-ph]].
- [27] S. Shirai, M. Yamazaki and K. Yonekura, arXiv:1003.3155 [hep-ph].
- [28] C. Cheung, A. L. Fitzpatrick and D. Shih, JHEP **0807**, 054 (2008)
[arXiv:0710.3585 [hep-ph]].
- [29] P. Meade, N. Seiberg and D. Shih, Prog. Theor. Phys. Suppl. **177**, 143 (2009)
[arXiv:0801.3278 [hep-ph]].

- [30] H. Ooguri, Y. Ookouchi, C. S. Park and J. Song, Nucl. Phys. B **808** (2009) 121 [arXiv:0806.4733 [hep-th]].
- [31] E. Poppitz and S. P. Trivedi, Phys. Lett. B **401**, 38 (1997) [arXiv:hep-ph/9703246].
- [32] F. Ferrari and A. Bilal, Nucl. Phys. B **469**, 387 (1996) [arXiv:hep-th/9602082];
A. Bilal and F. Ferrari, Nucl. Phys. B **480**, 589 (1996) [arXiv:hep-th/9605101].